Problem 1. $S$ is the set of all $(h, k)$ with $h, k$ non-negative integers such that $h + k < n$. Each element of $S$ is colored red or blue, so that if $(h, k)$ is red and $h' \leq h, k' \leq k$, then $(h', k')$ is also red. A type 1 subset of $S$ has $n$ blue elements with different first member and a type 2 subset of $S$ has $n$ blue elements with different second member. Show that there are the same number of type 1 and type 2 subsets.

Problem 2. $BC$ is a diameter of a circle center $O$. $A$ is any point on the circle with $\angle AOC > 60^\circ$. $EF$ is the chord which is the perpendicular bisector of $AO$. $D$ is the midpoint of the minor arc $AB$. The line through $O$ parallel to $AD$ meets $AC$ at $J$. Show that $J$ is the incenter of triangle $CEF$.

Problem 3. Find all pairs of integers $m > 2, n > 2$ such that there are infinitely many positive integers $k$ for which $k^n + k^2 - 1$ divides $k^m + k - 1$.

Problem 4. The positive divisors of the integer $n > 1$ are $d_1 < d_2 < \ldots < d_k$, so that $d_1 = 1, d_k = n$. Let $d = d_1d_2 + d_2d_3 + \cdots + d_{k-1}d_k$. Show that $d < n^2$ and find all $n$ for which $d$ divides $n^2$.

Problem 5. Find all real-valued functions on the reals such that $(f(x) + f(y))(f(u) + f(v)) = f(xu - yv) + f(xv + yu)$ for all $x, y, u, v$.

Problem 6. $n > 2$ circles of radius 1 are drawn in the plane so that no line meets more than two of the circles. Their centers are $O_1, O_2, \ldots, O_n$. Show that $\sum_{i<j} 1/O_iO_j \leq (n - 1)\pi/4$. 